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IN AN UNTAPERED WIGGLER FREE-ELECTRON-LASER AMPLIFIER

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SCALING LAWS FOR SATURATION INTENSITY AND LENGTH IN AN UNTAPERED WIGGLER FREE-ELECTRON-LASER AMPLIFIER¹

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Abstract: A combination of dimensional analysis and numerical computation methods was used to derive scaling laws for saturation intensity and saturation length in a millimeter wave free-electron-laser amplifier with an untapered wiggler.²

Key words: saturation, scaling laws, free-electron laser (FEL)

1. Introduction

Free-electron laser fundamental mode radiation must pass through linear and non-linear stages to reach saturation. High saturation intensity P_{ss} (the subscript SS means saturation value) is one of the most sought-after goals. Saturation intensity is related to linear stage growth rate (abbreviated to growth rate here). Research shows that when growth rate is high, P_{ss} is not necessarily high, because P_{ss} is related to both growth rate and saturation length z_{ss} (the axial distance that fundamental mode radiation must travel to reach saturation). Under conditions approximating one dimension, the physical parameters that determine P_{ss} and z_{ss} of untapered free-electron-laser amplifiers are: incident electron beam current I , relativity factor γ_0 , and relative energy dispersion $\Delta\gamma_0/\gamma_0$; wiggler magnetic field strength B_w and period λ_w ; fundamental mode laser wavelength λ_s and original fundamental mode radiation power P_{s0} . Of these, γ_0 , B_w , λ_w , and λ_s satisfy a resonance relationship – see

¹ This is the English title given by the Chinese authors. A more literal translation of the Chinese title yields "Scaling laws for saturation power of amplifier-type free-electron lasers." Note that the original Chinese title says "amplifier-type FEL" rather than "FEL amplifier."

² *Changcanshu*, which the authors translate as "untapered," can also be translated as "invariable."

formula (7). Because the complete expression of the dependence of P_{ss} and z_{ss} on the above physical parameters is a non-linear problem, it is difficult to use total analysis methods on it. In addition, there are many physical parameters, so even if $\Delta\gamma_0/\gamma_0$ is given, there are also the four independent fundamental physical parameters I , γ_0 , B_w , and γ_w . Thus, it is difficult to use a pure numerical fitting method to provide an expression.

Some newer, better methods that use a combination of dimensional analysis and numerical simulation are the United States' LLNL's³ Garrison-Wong^[1] (hereafter referred to as G-W) and Kumada-Sessler^[2] methods. These proceed from the Liouville light field equation group, and use the characteristic intensity and characteristic length of R. Banifacio's dimensional analysis method to nondimensionalize groups of equations. The nondimensional saturation intensity and saturation length they obtain are two nondimensional parameters (which are combinations of the above four physical parameters) and functions of the nondimensional parameters that describe the original conditions (document [1] assumes that energy dispersion is zero); afterwards, under conditions where $\lambda_s = 1\mu\text{m}$ and $10\mu\text{m}$, they use numerical calculation to approximately determine these two functions. We used the G-W method to derive scaling laws for saturation intensity under millimeter wave conditions, but our results were not as good as those attained in document [1] under conditions where $\lambda_s = 1\mu\text{m}$ and $10\mu\text{m}$.

This article proceeds from a group of one-dimensional fundamental equations of single particle theory. The most appropriate characteristic intensity and characteristic length were chosen to nondimensionalize this group of equations. Results proved that nondimensionalized single particle theory equation groups obtained in this way included only one nondimensional parameter, which is one fewer than the G-W method's nondimensional Liouville light field equations, and that there were also only two nondimensional parameters that described the original conditions: nondimensional original intensity P_{s0} and $\Delta\gamma_0/\gamma_0$. In this way, when $\Delta\gamma_0/\gamma_0$ is given, nondimensional saturation intensity and saturation length depend on just two nondimensional parameters. Based on these results, it is easy to use numerical calculation to approximately determine relatively simple scaling laws for the P_{ss} and z_{ss} of an untapered FEL amplifier with a certain wavelength. This article uses this method to give scaling laws for the P_{ss} and z_{ss} of millimeter wavelengths.

³ Lawrence Livermore National Laboratory

2. Groups of nondimensional parameters that determine nondimensional saturation intensity (P_{ss}/P_{s0}) and saturation length (z_{ss}/λ_w)

2.1 Nondimensionalization of fundamental equation groups

Single particle theory was used. At this point, the one-dimensional fundamental equation group for untapered wiggler free-electron lasers (ignoring the space charge effect) was the electron energy and phase equation from document [3] (here, the subscript j indicates the j th electron)

$$\frac{d\gamma_j}{dz} = -\frac{\alpha_1 e B_w}{\gamma_j m c^2 k_w} e_s \sin \psi_j, \quad e_s = \frac{e E_s}{m c^2} \quad (1)$$

$$\frac{d\psi_j}{dz} = k_w - \frac{k_s}{2\gamma_j^2} \left(1 + \frac{\alpha_1 e^2 B_w}{m^2 c^4 k_w^2}\right) + \frac{\alpha_2 e B_w}{\gamma_j^2 m c^2 k_w} e_s \cos \psi_j + \frac{d\phi}{dz} \quad (2)$$

$$\psi_j = (k_w + k_s)z - \omega_s t + \phi + \psi_{0j}$$

The amplitude and phase equations of the laser electric field are

$$\frac{de_s}{dz} = \frac{2\pi e^2 B_w I}{m^2 c^5 k_w} \cdot \left\langle \frac{\sin \psi_j}{\gamma_j} \right\rangle, \quad I = n_0 e c \quad (3)$$

$$\frac{d\phi}{dz} = \frac{2\pi e^2 B_w I}{m^2 c^5 k_w} \cdot \frac{1}{e_s} \cdot \left\langle \frac{\cos \psi_j}{\gamma_j} \right\rangle \quad (4)$$

In the above formulas, e_s is the reduction amplitude of the laser electric field, ϕ is the phase of the light field, k_w and k_s are, respectively, the wave numbers of the wiggler and the fundamental mode laser, and I is the current of the electron beam. For the circular polarization and linear polarization wigglers, α_1 is, respectively, 1 and 0.5; α_2 is, respectively, ± 1 and 0.5.

Nondimensional electric field

$$\bar{e}_s = \frac{L_1^2 e B_w}{\gamma_0^2 m c^2} e_s \quad (5)$$

and nondimensional length

$$\bar{z} = \frac{z}{L_2} \quad (6)$$

are entered. Of these parameters, L_1 and L_2 are the constants of length dimension. At this point, equations (1) through (4) can be nondimensionalized to

$$\frac{d\gamma_j}{dz} = - \frac{L_2 \alpha_1 \gamma_0^2}{L_1^2 k_w \gamma_j} \bar{e}_s \sin \psi_j, \quad (1')$$

$$\frac{d\psi_j}{dz} = L_2 k_w - \frac{L_2}{2\gamma_j^2} k_w \left(1 + \frac{\alpha_1 e^2 B_w^2}{m^2 c^4 k_w^2} \right) + \frac{L_2 \alpha_2 \gamma_0^2}{L_1^2 k_w \gamma_j^2} \bar{e}_s \cos \psi_j + \frac{d\phi}{dz} \quad (2')$$

$$\frac{d\bar{e}_s}{dz} = \frac{1}{c} \left(\frac{e}{mc^2} \right)^3 \frac{L_1^2 L_2 \lambda_w B_w^2 I}{\gamma_0^3} \left\langle \frac{\gamma_0}{\gamma_j} \sin \psi_j \right\rangle \quad (3')$$

$$\frac{d\phi}{dz} = \frac{1}{c} \left(\frac{e}{mc^2} \right)^3 \frac{L_1^2 L_2 \lambda_w B_w^2 I}{\gamma_0^3} \frac{1}{e_s} \left\langle \frac{\gamma_0}{\gamma_j} \cos \psi_j \right\rangle \quad (4')$$

2.2 Groups of nondimensional parameters that determine P_{SS}/P_{S0} and z_{SS}/λ_w

From formulas (1') through (4'), it is clear that when $L_1 = L_2 = \lambda_w$ and the following resonance relationship is used,

$$k_w \left(1 + \frac{\alpha_1 e^2 B_w^2}{m^2 c^4 k_w^2} \right) = 2k_w \gamma_0^2 \quad (7)$$

and γ_j in (1') through (4') appears in the form of γ_j/γ_0 , (1') through (4') can therefore be written as

$$\frac{d(\gamma_j/\gamma_0)^2}{dz} = - \frac{\alpha_1}{\pi} \bar{e}_s \sin \psi_j, \quad (1'')$$

$$\frac{d\psi_j}{dz} = 2\pi \left[1 - \left(\frac{\gamma_0}{\gamma_j} \right)^2 \right] + \frac{\alpha_2}{2\pi} \left(\frac{\gamma_0}{\gamma_j} \right)^2 \bar{e}_s \cos \psi_j + \frac{d\phi}{dz} \quad (2'')$$

$$\frac{d\bar{e}_s}{dz} = \epsilon \left\langle \frac{\gamma_0}{\gamma_j} \sin \psi_j \right\rangle \quad (3'')$$

$$\frac{d\phi}{dz} = \frac{\epsilon}{e_s} \left\langle \frac{\gamma_0}{\gamma_j} \cos \psi_j \right\rangle \quad (4'')$$

$$\gamma \equiv \frac{1}{c} \left(\frac{e}{mc^2} \right)^3 \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} \quad (8)$$

Equation groups (1'') through (4'') are equation groups of nondimensional variables γ_j/γ_0 , ψ_j , \bar{e}_s , and ϕ . They only contain one nondimensional coefficient, ϵ , and no longer contain γ_0 alone. The original value of γ_j/γ_0 is determined by relativistic energy dispersion $\Delta\gamma_0/\gamma_0$:

the original value of ψ_j is a constant that is unrelated to the parameters I , λ_w , B_w , λ_s , and γ_0 . The original value of ϕ is zero, and the original value of \bar{e}_s is \bar{e}_{s0} . The formula for e_{s0} is

$$\bar{e}_{s0} = \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0} \quad (9)$$

In the above formula, e_{s0} is the original value of e_s .

The above deductions prove that the nondimensional variables γ_j/γ_0 , ψ_j , \bar{e}_s , and ϕ are merely functions of z , ϵ , $\Delta\gamma_0/\gamma_0$, and $(e/mc^2)(\lambda_w^2 B_w/\gamma_0^2)e_{s0}$. In particular, $\bar{e}_s(\bar{z})$ can be written as

$$\bar{e}_s(\bar{z}) = F(\bar{z}, \epsilon, \frac{\Delta\gamma_0}{\gamma_0}, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0}), \quad \bar{z} = \frac{z}{\lambda_w} \quad (10)$$

Because nondimensional saturation length \bar{z}_{ss} fulfills

$$\left. \frac{\partial F}{\partial \bar{z}} \right|_{\bar{z}=\bar{z}_{ss}} = 0$$

therefore, \bar{z}_{ss} is just a function of ϕ , $(e/mc^2)(\lambda_w^2 B_w/\gamma_0^2)e_{s0}$, and $\Delta\gamma/\gamma_0$. Note that

$$\begin{aligned} P_{ss} &= \frac{m^2 c^3}{8\pi e^2} e_{ss}^2, \quad P_{s0} = \frac{m^2 c^3}{8\pi e^2} e_{s0}^2 \\ \bar{e}_{ss} &= \bar{e}_s(\bar{z}_{ss}) = F(\bar{z}_{ss}, \phi, \frac{\Delta\gamma_0}{\gamma_0}, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0}) \\ e_{ss}^2 &= \frac{m^2 c^4}{e^2} \frac{\gamma_0^4}{\lambda_w^4 B_w^2} \bar{e}_{ss}^2 \end{aligned}$$

The following theorem has been proved.

Theorem 1: For untapered wiggler free-electron laser amplifiers,

$$P_{ss} = \frac{m^4 c^9}{8\pi e^4} \frac{\gamma_0^4}{\lambda_w^2 B_w^2} F_1(\epsilon, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0}, \frac{\Delta\gamma_0}{\gamma_0}) \quad (11)$$

$$z_{ss} = \lambda_w F_2(\epsilon, \frac{e}{mc^2} \frac{\lambda_w^2 B_w}{\gamma_0^2} e_{s0}, \frac{\Delta\gamma_0}{\gamma_0}) \quad (12)$$

In these formulas, F_1 and F_2 are functions of ϵ , $(e/mc^2)(\lambda_w^2 B_w/\gamma_0^2)e_{s0}$, and $\Delta\gamma_0/\gamma_0$.

Theorem 1 stands up to arbitrary choice of I , λ_w , B_w , λ_s , and γ_0 .

Theorem 2: For conditions where $\Delta\gamma_0=0$ and P_{ss} has approximately no relation to e_{s0} (the G-W scaling law was obtained under these two assumptions), by taking $F_1(\epsilon) \propto j^{4/3}$, the G-W scaling law is obtained.

Actually, at this time,

$$P_{ss} \propto \frac{\gamma_0^4}{\lambda_w^4 B_w^2} \left(\frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} \right)^{4/3} \propto \lambda_w^{2/3} B_w^{2/3} I^{4/3} \quad (13)$$

The above formula is the G-W scaling law.

3. Scaling laws for the saturation intensity and saturation length of millimeter wavelength free-electron lasers

According to Theorem 1 in part 2.2, using multiple approximating functions F_1 within certain parameter ranges, for given $\Delta\gamma_0/\gamma_0$ and P_{s0} , P_{ss} can be written as

$$P_{ss} = A \frac{\gamma_0^4}{\lambda_w^4 B_w^2} j^\alpha \left(\frac{\lambda_w^4 B_w^2}{\gamma_0^3} \right)^\beta \quad (14)$$

In this way, the numerical calculation results of three I , γ_0 , B_w , and λ_w groups with characteristic meaning (abbreviated to parameter groups) can be used to determine the constants A , α , and β in formula (14), and then the computational results of other parameter groups can be used to test the formula.

We used the above method on a millimeter wavelength parameter group range of $200 \leq I/A^\circ \text{cm}^{-2} \leq 1000$, $3.5 \leq \gamma_0 \leq 11.775$, $2.75 \leq \gamma_w/\text{cm} \leq 16.5$, $0.2168 \leq \lambda_s/\text{cm} \leq 2.168$, and $1329 \leq B_w/\text{Gs} \leq 1626$ with an Aurora-1 [Shuguang-1] parameter group ($I=500 \text{A}/\text{cm}^2$, $\gamma_0=7.85$, $\lambda_w=11 \text{cm}$, $B_w=4065 \text{Gs}$, and $\lambda_s=0.867 \text{cm}$) as its center, and used the calculated results of document [3] to derive the following scaling laws for saturation intensity and saturation length of untapered millimeter wavelength free-electron laser amplifiers. When $P_{s0}=0.5 \text{MW}/\text{cm}^2$ and $\Delta\gamma_0/\gamma_0=0.03$,

$$P_{ss}/MW = 3.672 \times 10^7 (\epsilon - 0.203) \left(\frac{(\lambda_w/\text{cm})^2 (B_w/\text{Gs})}{\gamma_0^2} \right)^{-1.5} \quad (15)$$

$$z_{ss}/\text{cm} = 39.4 (\lambda_w/\text{cm}) (\epsilon - 0.203) \left(\frac{(\lambda_w/\text{cm})^2 (B_w/\text{Gs})}{\gamma_0^2} \right)^{-0.21} \quad (16)$$

These two formulas conform well to the computational results of the 55 parameter groups within the above parameter group range. For the great majority, relative error was less than 16%, and for a small minority, it was less than 15%. The range of change in ϵ for these parameter groups was $0.492 \leq \epsilon \leq 16.7$. When I was low and λ_w was low, ϵ would be low, and at this time B_w was also low. In formulas (15) and (16), $\epsilon - 0.203$ was a correction made when ϵ was lower, while under general conditions, $(\epsilon - 0.203) \approx \epsilon$. Substituting $(\lambda_w^2 B_w / \gamma_0^2)$ from formula (16) into formula (15) yields

$$P_{ss}/MW = 1.473 \times 10^{-4} \left(\frac{z_{ss}}{\lambda_w} \right)^{7.143} (\epsilon - 0.203)^{2.857} \quad (17)$$

From the above formula, it is clear that when P_{ss} and $\Delta\gamma_0/\Delta_0$ are given, what determines P_{ss} is ϵ and z_{ss}/λ_w , not ϵ and z_{ss} .

According to document [4], growth rate $G(L)$ is as follows under low gain conditions,

$$G(L) = \ln \frac{P_s(L)}{P_{s0}} = - \frac{\omega_b^2 a_w^2 k_w L^3}{8 c^2 \gamma_0^3} (J_0 - J_1)^2 \frac{d^2}{d\Phi_L} \frac{\sin^2 \Phi_L}{d\Phi_L} \quad (18)$$

$$\Phi_L \equiv [k_w (1 - \frac{c}{v_b}) + k_w] \frac{L}{2}, \quad \omega_b^2 \equiv \frac{4\pi n_0 e^2}{m} \quad (19)$$

while the highest growth rate in high gain situations is

$$G(L) = \ln \frac{P_s(L)}{P_{s0}} = \sqrt{3} \sqrt{2} \left(\frac{\omega_b^2 a_w^2 k_w L^3}{8 c^2 \gamma_0^3} \right)^{1/3} \quad (19)$$

Note that

$$g(L) \equiv \frac{\omega_b^2 a_w^2 k_w L^3}{8 c^2 \gamma_0^3} = \frac{1}{8} \frac{1}{c} \left(\frac{e}{mc^2} \right)^3 \frac{\lambda_w B_w^2 I L^3}{\gamma_0^3}$$

but when $L = \lambda_w$,

$$g(\lambda_w) = \frac{1}{8} \frac{1}{c} \left(\frac{e}{mc^2} \right)^3 \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} \quad (20)$$

As a result,

$$\epsilon = 8g(\lambda_w) \quad (21)$$

Therefore, ϵ is a physical quantity that describes the growth rate where $z = \lambda_w$. In this way, according to the theorems in part 2, when P_{s0} and $\Delta\gamma_0/\gamma_0$ are given, P_{ss} is determined by and only determined by growth rate j and z_{ss}/λ_w ; the quantitative relationship of millimeter wavelengths is similar to formula (17). It is thus clear that nondimensional saturation length has a large influence on P_{ss} .

When current I is expressed in units of A/cm² and other physical quantities are expressed in Gaussian units,

$$\epsilon \equiv \frac{1}{c} \left(\frac{e}{mc^2} \right)^2 \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3} = 0.2 \times 10^{-10} \frac{\lambda_w^4 B_w^2 I}{\gamma_0^3}$$

From formula (15), when $\epsilon \gg 0.203$, the following simplified formula can yield P_{ss}

$$\begin{aligned} P_{ss} &= 0.7344 \times 10^{-3} \cdot \epsilon \cdot \lambda_w^{-3} B_w^{-1.5} \gamma_0^3 = 0.7344 \times 10^{-3} \lambda_w B_w^{0.5} I \\ &= 0.7344 (\lambda_w^4 B_w^2 I^4)^{1/4} \end{aligned} \quad (22)$$

But when j is less than 0.5, the error from simplified formula (22) may be very large.

On the basis of the preceding theorems and formula (15), it is possible to find the dependence of P_{ss} on P_{s0}

$$P_{ss} = 4.36677 \times 10^7 \cdot P_{s0}^{0.25} \cdot (\epsilon - 0.203) \cdot \left(\frac{\lambda_w^2 B_w}{\gamma_0^2} \right)^{-1.5} \quad (23)$$

We calculated the above 55 parameter groups when $P_{s0} = 1 \text{ MW/cm}^2$, and as a result, when $P_{s0} = 1 \text{ MW/cm}^2$, P_{ss} was 1 to 1.2 times ($2^{0.25} = 1.789$) greater than when $P_{s0} = 0.5 \text{ MW/cm}^2$. More than half [of the P_{ss}] were more than 1.1 times greater, and where current I was less than 300A, $[P_{ss}]$ was less than 1.05 times greater. Of course, when there are large variations in P_{s0} , the dependence of P_{ss} on $(\lambda_w^2 B_w / \gamma_0) e_{s0}$ will change, and the dependence of P_{ss} on P_{s0} will thus change.

Below are given comparisons of groups of numerical calculation results of P_{ss} and z_{ss} of parameter groups with characteristic meaning and comparisons of calculation results of formulas (15) and (16), where $P_{s0} = 0.5 \text{ MW/cm}^2$ and $\Delta\gamma_0/\gamma_0 = 0.03$. At the same time, these

numerical calculation results are used to explain several physics problems.

(1) $\gamma_0 = 7.85$, $\lambda_w = 11\text{cm}$, $B_w = 4065\text{Gs}$, $\lambda_s = 0.867\text{cm}$, ⁽¹⁾电流 I 改变

$I (\text{A/cm}^2)$	1000	625	500	300	200
$P_{ss} (\text{MW/cm}^2)$	532.8	311.4	257.4	141.2	80.3
$\frac{z_{ss}}{\lambda_w}$	3.271	3.681	4.004	4.5001	4.91
$\varepsilon - 0.203$	9.8	6.049	4.796	2.796	1.796
$P_{ss}^{(1)}$	504.61	311.49	246.95	143.99	92.51
$(\frac{z_{ss}}{\lambda_w})^{(1)}$	3.326	3.774	3.98	4.548	5.036

Key: (1) Current I varies

Of the above values, P_{ss} and z_{ss}/λ_w are calculated values from document [3], and $P_{ss}^{(1)}$ and $(z_{ss}/\lambda_w)^{(1)}$ are values given by formulas (15) and (16).

(2) $\gamma_0 = 7.85$, $\lambda_w = 11\text{cm}$, $B_w = 1646\text{Gs}$, $\lambda_s = 0.2168\text{cm}$, ⁽¹⁾电流 I 改变

$I (\text{A/cm}^2)$	625	500	300
$P_{ss} (\text{MW/cm}^2)$	155.8	121.5	57.9
$\frac{z_{ss}}{\lambda_w}$	7.637	8.182	9.82
$\varepsilon - 0.203$	0.822	0.617	0.289
$P_{ss}^{(1)}$	164.3	123.3	57.8
$(\frac{z_{ss}}{\lambda_w})^{(1)}$	7.620	8.200	9.996

(3) $\gamma_0 = 7.85$, $\lambda_w = 5.5\text{cm}$, $B_w = 8129\text{Gs}$, $\lambda_s = 0.4335\text{cm}$, 电流 I 改变

$I (\text{A/cm}^2)$	625	500	300
$P_{ss} (\text{MW/cm}^2)$	197.8	150.8	72.17
$\frac{z_{ss}}{\lambda_w}$	6.28	6.636	7.909
$\varepsilon - 0.203$	1.36	1.047	0.547
$P_{ss}^{(1)}$	198.1	152.51	79.68
$(\frac{z_{ss}}{\lambda_w})^{(1)}$	6.37	6.82	8.08

Key: (1) Current I varies

The above three groups of results were [obtained] under conditions of different λ_s and λ_w when studying the dependence of P_{ss} and z_{ss}/λ_w on ϵ . Clearly, when $\epsilon \gg 0.203$, P_{ss} is truly approximately proportional to j , and when ϵ is smaller, P_{ss} is approximately proportional to $\epsilon - 0.203$. We also used data from these three groups to determine the degree of $\epsilon - 0.203$ in formula (16).

4. $I = 625 \text{ A/cm}^2$, $\gamma_0 = 7.85$, $\lambda_s = 0.867 \text{ cm}$, λ_w 改变

λ_w (cm)	16.5	14	11	8	5.5
B_w (Gs)	2148	2786	4065	6655	11820
P_{ss} (MW/cm ²)	339.1	326.1	311.4	301.9	283.3
$\frac{z_{ss}}{\lambda_w}$	3.242	3.394	3.681	4.183	4.729
$\epsilon - 0.203$	8.635	7.503	6.049	4.483	3.143
$P_{ss}^{(1)}$	342.95	330.35	311.5	286.53	261.65
$(\frac{z_{ss}}{\lambda_w})^{(1)}$	3.287	3.461	3.774	4.166	4.741

Key: (1) λ_w varies

We used the preceding group of results to determine the degrees and coefficients of $\lambda_w^2 B_w / \gamma_0^2$ in formulas (15) and (16). From this group of results, one can see that when I , γ_0 , and λ_s do not change and λ_w is reduced, although B_w increases a great deal, ϵ still decreases a great deal, so that even if z_{ss}/λ_w rises, P_{ss} still falls.

(5) $I = 625 \text{ A/cm}^2$, $\gamma_0 = 7.85$, λ_s 和 λ_w 成比例缩小

λ_w (cm)	11	5.5	2.75
B_w (Gs)	4065	8129	16260
λ_s (cm)	0.867	0.4335	0.2168
P_{ss} (MW/cm ²)	311.4	197.8	87.9
$\frac{z_{ss}}{\lambda_w}$	3.681	6.18	12.2
$\epsilon - 0.203$	6.049	1.36	0.188
$P_{ss}^{(1)}$	311.5	198.2	77.35
$(\frac{z_{ss}}{\lambda_w})^{(1)}$	3.77	6.37	12.38

Key: (1) λ_w and λ_s are reduced proportionately

The group of results above shows that when I and γ_0 do not change and λ_w and λ_s are reduced proportionately, the results are similar to those from group 4: even though B_w increases a great deal, j decreases very much, so that even if z_{ss}/λ_w increases, P_{ss} still decreases by a large margin.

(6) $\gamma_0 = 3.92$, $\lambda_w = 2.75\text{cm}$, $B_w = 16260\text{Gs}$, $\lambda_s = 0.867\text{cm}$, ⁽¹⁾ I 改变

$I (\text{A/cm}^2)$	625	500	300
$P_{ss} (\text{MW/cm}^2)$	148.2	110.3	54.19
$\frac{z_{ss}}{\lambda_w}$	4.546	4.729	5.454
$\epsilon - 0.203$	2.927	2.301	1.299
$P_{ss}^{(1)}$	150.43	118.25	66.78
$(\frac{z_{ss}}{\lambda_w})^{(1)}$	4.527	4.82	5.592

Key: (1) I varies

The preceding is a miniaturized wiggler parameter group where λ_s stays at 0.867 cm and B_w is increased, while λ_w and γ_0 are simultaneously decreased. Compared with the results from the first group, P_{ss} decreased by a large margin, because although z_{ss}/λ_w increased, $\epsilon - 0.203$ declined a great deal. Compared with the fifth parameter group, when γ_0 increased from 3.92 to 7.85, the P_{ss} of $I=300$ increased from 54.19 to 87.9.

(7) $\gamma_0 = 11.775$, $\lambda_w = 11\text{cm}$, $B_w = 2910\text{Gs}$, $\lambda_s = 0.2168\text{cm}$, ⁽¹⁾ I 改变

$I (\text{A/cm}^2)$	625	500	300
$P_{ss} (\text{MW/cm}^2)$	208.5	136.9	81.84
$\frac{z_{ss}}{\lambda_w}$	8.09	8.77	10.6
$\epsilon - 0.203$	0.746	0.556	0.253
$P_{ss}^{(1)}$	214.01	159.56	72.5
$(\frac{z_{ss}}{\lambda_w})^{(1)}$	8.219	8.870	10.89

Key: (1) I varies

The preceding is a parameter group where γ_0 is increased while B_w is decreased to reduce λ_s even more. Compared with the results of the first group, this happened because ϵ fell

by a large margin. Although z_{ss}/λ_w increased, P_{ss} still declined a great deal.

Finally, we want to point out that from the computational results given above, it is possible to find many examples where ϵ is low, whereas z_{ss}/λ_w is high, and thus P_{ss} is [also] high. For example, compare group number 5, where $\lambda_w=2.75\text{cm}$, and group number 2, where $I=300\text{A}/\text{cm}^2$.

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